PHYSICAL REVIEW E VOLUME 61, NUMBER 6 JUNE 2000

Multiscaling of energy correlations in the random-bond Potts model

Jesper Lykke Jacobsen
LPTMS, Bâtiment 100, Université Paris–Sud, F-91405 Orsay, France
(Received 16 December 1999)

We numerically calculate the exponents governing the disorder averaged and fixed-sample decay of the energy-energy correlator in the q-state random-bond Potts model. Our results are in good agreement with a two-loop expansion around $q=2$ recently found from perturbative renormalization group techniques, fulfill the correlation length bound $\nu \geqslant 2/d$, and give further evidence against replica symmetry breaking in this class of models.

PACS number(s): 05.70.Jk, 64.60.Ak, 64.60.Fr

I. INTRODUCTION

The q-state random-bond Potts model is an interesting framework for examining how a phase transition is modified by quenched disorder coupling to the local energy density. For $q>2$ such randomness acts as a relevant perturbation [1], and for $q>4$ it even changes the nature of the transition from first to second order (see Ref. [2] for a review). In the regime where $(q-2)$ is small, a score of analytical results have been obtained from the perturbative renormalization group, and the various expansions for the central charge and the multiscaling exponents for the moments of the spin-spin correlator compare convincingly to recent numerical work [3].

A particularly useful way of carrying out these simulations is to consider the finite-size scaling of the Lyapunov spectrum of the (random) transfer matrix, thus generalizing the method commonly applied to the eigenvalue spectrum in a pure system [4]. A definite advantage over the more traditional technique of Monte Carlo simulations [5] is that the transfer matrices allow for a representation in which q can be regarded as a continuously varying parameter [6,4], and in particular one can study small non-integer values of $(q-2)$.

The outcome of applying this method to the *energetic* sector of the transfer matrix, however, led to contradictory results [4]. Most notably, the exponent $\tilde{X}_1$ describing the asymptotic decay of the disorder-averaged first moment of the two-point function $\overline{\langle \varepsilon(x_1)\varepsilon(x_2)\rangle} \propto |x_1-x_2|^{-2\tilde{X}_1}$ seemed to be a rapidly decreasing function of q, in sharp disagreement with an exact bound on the correlation length exponent, $\nu \geqslant 2/d$ [7], which in our notation reads $\tilde{X}_1 \geqslant 1$.

More recent numerical work has emphasized the importance of crossover behavior [5] from the random fixed point to, on one side, the pure Potts model, and on the other, a percolationlike limit [4] in which the ratio $R=K_1/K_2$ between strong and weak couplings tends to infinity. It became clear that while the fixed ratio $R=2$ employed in Ref. [4] seems to have been adequate for studying the spin sector when $(q-2)$ is small, in general higher values of R are needed to measure the true random behavior in the regime $q>4$ [5].

These findings were put on a firmer ground when it was realized [8] that Zamolodchikov's c-theorem [9] can be used to explicitly trace out the critical disorder strength $R_*(q)$ as a function of q, by scanning for an extremum of the effective central charge. In conjunction with an improved transfer matrix algorithm in which the Potts model is treated through its representation as a loop model, this allowed Jacobsen and Picco [8] to produce very accurate results for the central charge and the magnetic scaling dimension in the regime $q \gg 4$.

On the analytical side, the perturbative expansions for the first three moments of the energetic two-point function have been known for quite some time [10]. It was, however, only very recently that Jeng and Ludwig [11] succeded in generalizing these computations to a general Nth moment of the energy operator $\overline{\langle \varepsilon(x_1)\varepsilon(x_2)\rangle^N} \propto |x_1-x_2|^{-2\tilde{X}_N}$, yielding

$$\tilde{X}_N = N\left(1-\frac{2}{9\pi^2}(3N-4)(q-2)^2+O(q-2)^3\right). \quad (1)$$

In particular, this makes available the experimentally relevant exponent $\tilde{X}_0'$ describing the typical decay of the energy-energy correlator in a fixed sample at criticality [10].

In this Rapid Communication we show that by combining the methods of Refs. [4,8] the exponents $\tilde{X}_1$ and $\tilde{X}_0'$ can be quite accurately determined numerically for small $(q-2)$. In particular, we find $\tilde{X}_1 \geqslant 1$ in full agreement with the correlation length bound [7], and our results lend strong support to the above two-loop results of the perturbative renormalization group.

II. SIMULATIONS

In order to compare our results with those of the $(q-2)$ expansion, while on the other hand staying comfortably away from $q=2$ where logarithmic corrections are expected [12], our main series of data has $q=2.5$. Iterating the transfer matrix for a strip of width L a large number $M \gg L$ of times, we examine the probability distribution of the ratio between the two largest Lyapunov exponents Λ_0, Λ_1 [13] in terms of the free energy gap $\Delta f(L)=(1/LM)\ln(\Lambda_0/\Lambda_1)$. We employ the loop representation of the transfer matrix where each loop on the surrounding lattice is given a weight $n=\sqrt{q}$ [14], and bond randomness is incorporated by weighing the two possible vertex configurations by w_i and $1/w_i$, where w_i is a quenched random variable that can assume two different values s and $1/s$, each one with probability 1/2 [8]. By construc-

TABLE I. Effective central charge $c(L,L+2)$ of the $q=2.5$ state model, as a function of disorder strength s and the strip width L.

s	$c(6,8)$	$c(8,10)$	$c(10,12)$
1.0	0.637 75	0.648 44	0.654 04
1.5	0.637 74	0.648 54	
2.0	0.637 70	0.648 74	
2.1	0.637 67	0.648 77	
2.2	0.637 64	0.648 79	0.654 15
2.3	0.637 59	0.648 80	0.654 15
2.4	0.637 54	0.648 79	0.654 13
2.5	0.637 48	0.648 79	0.654 11
2.6	0.637 40	0.648 77	0.654 09

tion, the system is then on average at its self-dual point [15]. The strength of the disorder is measured by $s>1$, which is related to the ratio between strong and weak bonds by $R=K_1/K_2=\ln(1+s\sqrt{q})/\ln(1+\sqrt{q}/s)$. The maximum strip width employed in the study is $L_{\max}=12$.

Following Ref. [8], we start by locating the critical disorder strength s_* by searching for a maximum of the effective central charge. To do so, we must be able to determine finite-size estimates $c(L,L+2)$ [16] with five significant digits, which means that the free energy $f_0(L)=(1/LM)\ln(\Lambda_0)$ must be known with seven digit precision. These considerations fix the necessary number of iterations to be $M=10^8$.

Our results for $c(L,L+2)$ as a function of L and s are shown in Table I. For a sufficiently large system size L, these data exhibit a maximum as a function of s, the position of which determines a finite-size estimate $s_*(L)$, which converges to s_* as $L\to\infty$. From the data of Table I, supplemented by improved three-point fits [4] (not shown), we extrapolate to $s_*(q=2.5)=2.5(1)$.

The fluctuations in $\Delta f(L)$ are examined by dividing the strip into M/m samples, each one of length $m=10^5$, from which the first few cumulants of $\Delta f(L)$ can be determined. As discussed in Ref. [4], the exponent $\tilde{X}_0'$ is related to the finite-size scaling [17] of the mean value (first cumulant) of $\Delta f(L)$, whereas $\tilde{X}_1$ is similarly determined from the sum of the entire cumulant expansion. In practice, the second cumulant is roughly two orders of magnetude smaller than the first, and higher cumulants are expected to be further suppressed, even though their determination is made difficult by

TABLE II. Effective exponent $\tilde{X}_0'(L)$ of the $q=2.5$ state model, as a function of disorder strength s and the strip width L.

s	$\tilde{X}_0'(6)$	$\tilde{X}_0'(8)$	$\tilde{X}_0'(10)$	$\tilde{X}_0'(12)$
1.0	0.9791	0.9513	0.9375	0.9293
1.5	1.0026	0.9754	0.9622	
2.0	1.0468	1.0191	1.0057	
2.2	1.0662	1.0377	1.0238	1.0158
2.3	1.0760	1.0470	1.0328	1.0246
2.4	1.0859	1.0563	1.0418	1.0333
2.5	1.0958	1.0656	1.0506	1.0419
2.6	1.1057	1.0749	1.0595	1.0504

TABLE III. Effective exponent $\tilde{X}_1(L)$ of the $q=2.5$ state model, as a function of disorder strength s and the strip width L.

s	$\tilde{X}_1(6)$	$\tilde{X}_1(8)$	$\tilde{X}_1(10)$	$\tilde{X}_1(12)$
1.0	0.9791	0.9513	0.9375	0.9293
1.5	1.0016	0.9746	0.9614	
2.0	1.0389	1.0124	0.9997	
2.2	1.0534	1.0269	1.0142	1.0065
2.3	1.0603	1.0338	1.0212	1.0133
2.4	1.0671	1.0405	1.0279	1.0198
2.5	1.0735	1.0469	1.0343	1.0260
2.6	1.0798	1.0531	1.0405	1.0320

numerical instabilities. We can therefore with confidence truncate the sum of the cumulants after the second one.

Resulting finite-size estimates [17] of $\tilde{X}_0'$ and $\tilde{X}_1$ are shown in Tables II and III, respectively. Unlike what seemed to be the situation in the *magnetic* sector, these estimates exhibit a pronounced dependence on s. Jacobsen and Cardy [4] worked at fixed $R=2$, which for $q=2.5$ would correspond to $s\simeq 1.7$, and found $\tilde{X}_0'<1$ for all $q>2$. We see here that the correct way to extract these exponents is to extrapolate the $s=s_*$ data to the $L\to\infty$ limit. With the help of improved two-point estimates [4] (not shown), we thus obtain

$$\tilde{X}_0'=1.02(1), \quad \tilde{X}_1=1.00(1), \tag{2}$$

which verifies the bound of Ref. [7]. These exponents, as well as the result for their difference $\tilde{X}_0'-\tilde{X}_1=0.015(5)$, are in very good agreement with the $(q-2)$-expansion; see Eq. (1).

We have also performed simulations for higher values of q, where the discrepancy between Refs. [4] and [7] was even more pronounced, since s_* is an increasing function of q. For $q=2.75$ and $q=3$, we had to increase the number of iterations to $M=10^9$ in order to keep the error bars under control despite the higher disorder strength. In all cases we found good agreement with Ref. [7] and with the $(q-2)$ expansion, at least in the range where the latter can be assumed to be valid. A summary of our results is given in Table IV.

III. CONCLUSION

In summary, we have shown that the apparent violation of the correlation length bound [7] observed in Ref. [4] can be

TABLE IV. Numerical results for the critical disorder strength (s_* or R_*) and energetic scaling exponents ($\tilde{X}_0'$ and $\tilde{X}_1$) as functions of q. The agreement with the two-loop expansion, Eq. (1), is good.

q	s_*	R_*	$\tilde{X}_0'$		$\tilde{X}_1$	
			Numerics	Theory	Numerics	Theory
2.50	2.5(1)	3.3(2)	1.02(1)	1.023	1.00(1)	1.006
2.75	3.0(3)	4.1(5)	1.04(2)	1.051	1.01(1)	1.013
3.00	3.5(5)	4.7(10)	1.06(3)	1.09	1.02(2)	1.02

dismissed as a crossover effect due to the lack of tuning to the critical disorder strength. In conjunction with the results on degeneracy and descendents given in Ref. [4], we would thus claim that the transfer matrix method [4,8] can, at least in principle, be used to relate the entire Lyapunov spectrum to the operator content of the (as yet unknown) underlying conformal field theory.

In particular, we have supplied convincing numerical validation of the two-loop expansion (1) for the energetic multiscaling exponents [11]. Our results also provide further evidence in favor of the replica symmetric approach to the perturbative calculations, since the assumption of initial replica symmetry breaking leads to $\tilde{X}_1 = 1 + O(q-2)^3$ [18], which seems to be ruled out by the results given in Table IV.

[1] A.B. Harris, J. Phys. A **7**, 1671 (1974).
[2] J. Cardy, Physica A **263**, 215 (1999).
[3] C. Chatelain and B. Berche, Nucl. Phys. B **572**, 626 (2000).
[4] J.L. Jacobsen and J. Cardy, Nucl. Phys. B **515** [FS], 701 (1998).
[5] M. Picco, e-print cond-mat/9802092.
[6] H.W.J. Blöte and M.P. Nightingale, Physica A **112**, 405 (1982).
[7] J.T. Chayes, L. Chayes, D.S. Fisher, and T. Spencer, Phys. Rev. Lett. **57**, 2999 (1986); Commun. Math. Phys. **120**, 501 (1989).
[8] J.L. Jacobsen and M. Picco, Phys. Rev. E **61**, 13 (2000).
[9] A.B. Zamolodchikov, Pis'ma Zh. Éksp. Teor. Fiz. **43**, 565 (1986) [JETP Lett. **43**, 730 (1986)].
[10] A.W.W. Ludwig, Nucl. Phys. B **330**, 639 (1990).
[11] M. Jeng and A.W.W. Ludwig, e-print cond-mat/9910181.
[12] J.L. Cardy, J. Phys. A **19**, L1093 (1986).
[13] H. Furstenberg, Trans. Am. Math. Soc. **68**, 377 (1963).
[14] R.J. Baxter, *Exactly Solved Models in Statistical Mechanics* (Academic Press, London, 1982).
[15] W. Kinzel and E. Domany, Phys. Rev. B **23**, 3421 (1981).
[16] H.W. Blöte, J.L. Cardy, and M.P. Nightingale, Phys. Rev. Lett. **56**, 742 (1986); I. Affleck, *ibid.* **56**, 746 (1986).
[17] J.L. Cardy, J. Phys. A **16**, L355 (1983).
[18] Vik.S. Dotsenko, Vl.S. Dotsenko, M. Picco, and P. Pujol, Europhys. Lett. **32**, 425 (1995).